Limitations of the Standard Method for the Midcourse Correction of Lunar Trajectories

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A method for computation of the midcourse correction velocity for lunar impact trajectories is analyzed. Only the magnitude of the impact miss, but not the impact time, is considered for correction. The method introduced in 1959 by W. Kizner utilizes the impact parameter as the measure of the target miss. It is shown that this method has certain limitations. When applied for correction of trajectories intended to impact a material point in space, or for correction of lunar fly-by trajectories, this method performs flawlessly. It also performs reasonably well when applied on standard trajectories. However, it may become inadequate for a general type of impact trajectory. The reasons for that are found to be inherent in the nature of the method. Two kinds of deficiencies are determined due to the method: the possible incorrect magnitude of the computed midcourse correction velocity, and the nonuniqueness in relation between the target miss and the injection errors or midcourse correction velocity. A partial improvement of the method is proposed. Also another measure of the target miss which does not cause drawbacks and which is also used presently, however less frequently, is picked up and recommended.

1. Introduction

WHEN a space vehicle is launched with the objective of impacting at a specific point on the lunar surface, it initially experiences thrust acceleration during the powered flight for a very short period of time and then coasts for a long time. The space vehicle is guided along a trajectory during the powered flight which is designed with our best present knowledge of vehicle dynamics and environmental forces. The alignment of the vehicle along the desired trajectory which is called nominal, is accomplished by means of vehicle steering. However, different errors cause trajectory deviations which may cause a significant target miss. Therefore, to correct this miss an additional velocity impulse, $\Delta\bar{V}_{mc}$, is required and applied approximately halfway between the earth and the moon. Determination of the necessary $\Delta\bar{V}_{mc}$ requires a computational method. Such a method which we will call the standard method was introduced by W. Kizner of the Jet Propulsion Lab. (JPL) in 1959 and is described in Refs. 2 and 3. It utilizes the vector-distance from the center of the target body to the incoming asymptote, the impact parameter, $\bar{B}$ (see Fig. 1) as the measure of the target miss. $\bar{B}$ is given by two components, $\bar{B}\cdot\hat{T}$† and $\bar{B}\cdot\hat{R}$ in the *TRS* coordinate system (see Fig. 2 and Appendix A). The method is used in practice for both fly-by and impact trajectories for preliminary studies.

Until now the $\bar{B}\cdot\hat{T}$ and $\bar{B}\cdot\hat{R}$ method is the most widely used method for representation of the target miss for preliminary studies. (Impact longitude and latitude and other parameters are also sometimes used.) Certain undesired peculiarities in ΔV_{mc} result due to this method; therefore the analysis of the standard method is the subject of this study.

2. Minimization of ΔV_{mc}

One single impulse is the only type of correction considered by this analysis. For t_4 (t = time; for subscripts, see Fig. 1) fixed the ΔV_{mc} vector is completely determined. If, however, the impact time is arbitrary, many different possibilities of $\Delta\bar{V}_{mc}$ maneuver exist. Only corrected trajectories requiring the smallest ΔV_{mc} are considered in this study. We presume that we know the nominal trajectory and position and velocity errors to be corrected.

Looking into Fig. 3 and calling $\bar{v}_2$ the perturbed velocity at point 2, we realize that conceptually the proper impact on the moving moon can be accomplished by two extremes and many intermediate methods of applying $\Delta\bar{V}_{mc}$ as: 1) $\Delta\bar{V}_a$ in direction of perturbed $\bar{v}_2$ only, 2) $\Delta\bar{V}_b$ normal to $\bar{v}_2$, 3) intermediate cases if, $\Delta\bar{V}_c = \alpha_1\cdot\Delta\bar{V}_a + \alpha_2\cdot\Delta\bar{V}_b$, where α_1 and α_2 are appropriate coefficients.

We will make a very simple model of the vehicle's and lunar motion. We will assume that both moon and vehicle move rectilinearly, on mutually perpendicular trajectories. Besides that we will assume that the moon is represented by the point mass and its velocity $\bar{v}_m$ is constant and known. Vehicle's velocity $\bar{V}_2$ is also considered constant and known.

Case A

Looking at Fig. 4a we see that the condition of collision can be described as

$$(t_{24} + \delta t_4) = S/v_m = R_{24}/(V_2 + \Delta V_a) \tag{1}$$

where δt_4 is perturbation in t_4, impact time. Therefore

$$\Delta V_a = (R_{24}v_m/S) - V_2 \tag{2}$$

Case B

This time we add $\Delta\bar{V}_b$ perpendicular to $\bar{V}_2$ in such a way that the vehicle will move along a new line intersecting the path of the moon at point 14, separated from point 4 by the length δS (see Fig. 4b);

$$t_{13-14} = (S + \delta S)/v_m \tag{3}$$

$$t_{2-14} = \frac{R_{2-14}}{(V_2^2 + \Delta V_b^2)^{1/2}} = \frac{(R_{24}^2 + \delta S^2)^{1/2}}{(V_2^2 + \Delta V_b^2)^{1/2}} \tag{4}$$

The condition for collision is that

$$t_{13-14} = t_{2-14} \tag{5}$$

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† Hat indicates the unit vector.

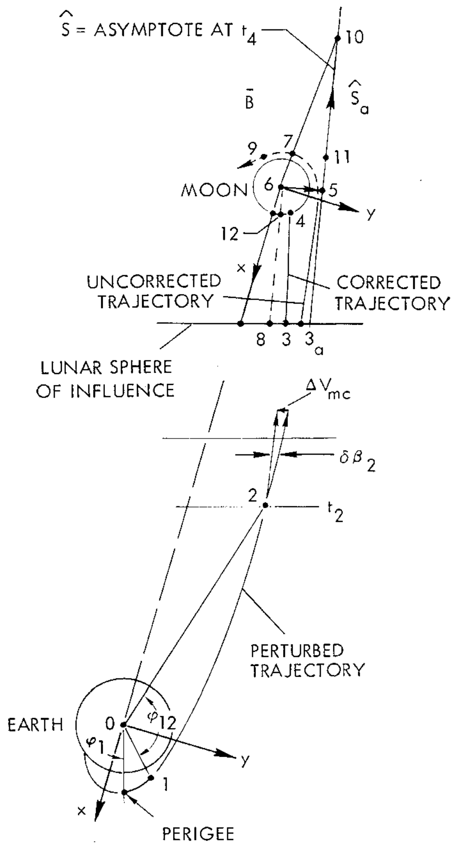
$$\frac{S+\delta S}{V_m} = \frac{(R_{24}{}^2 + \delta S^2)^{1/2}}{(V_2{}^2 + \Delta V_b{}^2)^{1/2}} = \frac{\left(R_{24}{}^2\left\{1+\left(\frac{\delta S}{R_{24}}\right)^2\right\}\right)^{1/2}}{\left(V_2{}^2\left\{1+\left(\frac{\Delta V_b}{V_2}\right)^2\right\}\right)^{1/2}} = \frac{R_{24}}{V_2} \quad (6)$$

since

$$\delta S/R_{24} = \Delta V_b/V_2 \quad (7)$$

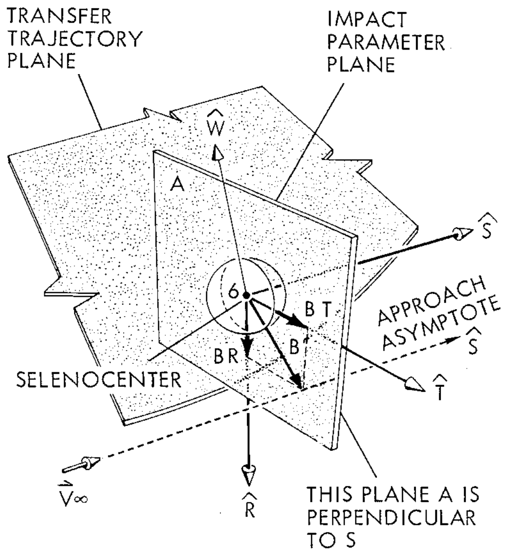
Therefore, using (6) and (7) obtain

$$\Delta V_b = \frac{\delta S}{R_{24}} V_2 = \frac{V_2}{R_{24}}\left(\frac{v_m}{V_2} R_{24} - S\right) = v_m - \frac{S}{R_{24}} V_2 \quad (8)$$



0 = CENTER OF THE EARTH
1 = INJECTION POINT
2 = MIDCOURSE CORRECTION POINT
3 = ENTRANCE INTO SELENO CENTRIC ZONE
4 = IMPACT ON LUNAR SURFACE
5 = END POINT OF B̄
6 = CENTER OF THE MOON, CM
7 = PERICYNTHION = DCA = r_p
= DISTANCE OF CLOSEST APPROACH
8 = ENTRANCE INTO SELENO CENTRIC ZONE ALONG A STANDARD TRAJECTORY
9 = POINT FOR δTF COMPUTATION
10 = ORIGIN OF ASYMPTOTES
11 = POINT ALONG Ŝ CORRESPONDING TO t_4
12 = IMPACT POINT ON LUNAR SURFACE FOR A STANDARD TRAJECTORY

Fig. 1 General nomenclature along the trajectory.



r_m = RADIUS VECTOR OF THE CENTER OF THE MOON MEASURED FROM EARTH CENTER
Ŵ = UNIT VECTOR PERPENDICULAR TO MOON'S ORBITAL PLANE
B̄S FORMS VEHICLE TRANSFER PLANE AT THE IMPACT POINT. B̂ IS NORMAL TO Ŝ
*SEE APPENDIX FOR DETAILS

$$\hat{W} = \frac{\vec{r}_m \times \dot{\vec{r}}_m}{|\vec{r}_m \times \dot{\vec{r}}_m|}$$

$$\hat{T} = \hat{S} \times \hat{W}$$

$$\hat{R} = \hat{S} \times \hat{T}$$

T = AXIS IS ALONG THE CROSSING OF THE A-PLANE AND THE MOON'S ORBITAL PLANE

Fig. 2 *TRS* coordinate system.

Case C

It can be shown by the method of Lagrange multiplier that the optimal policy would be ΔV_c for which following condition is satisfied:

$$\alpha_1 \Delta V_a / \alpha_2 \Delta V_b = S/R_{24} \quad (9)$$

For this optimal case

$$\Delta V_{\min} = \Delta V_c = (R_{24} v_m - S V_2)/[(R_{24}{}^2 + S^2)^{1/2}] \quad (10)$$

This means that $\alpha_1 = S^2/(R_{24}{}^2 + S^2)$, and $\alpha_2 = R_{24}{}^2/(R_{24}{}^2 + S^2)$.

Reference 1 presents more detailed analysis of this problem and considers more realistic cases.

3. Impact Parameter

If a trajectory is designed to impact a material point in space, then for its perturbed version missing this point and coasting around it in a conic-shape trajectory, the apparently simplest way to measure the miss would be to measure the vehicle's distance from that point at its closest approach, DCA.[2] This is the perifocal distance or peri-cynthion distance for lunar trajectories designated by r_p. See Fig. 5. Compute r_p in the inertial selenocentric coordinate system;

$$r_p = [\mu(e-1)]/v_\infty{}^2 \quad (11)$$

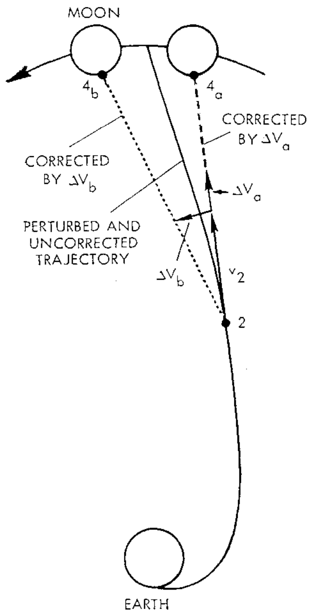


Fig. 3 Two possible methods of correction.

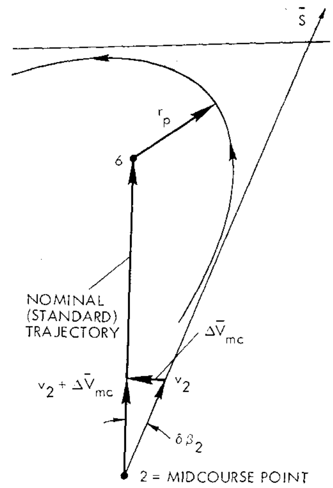


Fig. 5 Geometry of correction.

where orbital eccentricity

$$e = [1 + (C^2/\mu^2)h]^{1/2} \tag{12}$$

orbital energy

$$h = v^2 - 2\mu/r = v_\infty^2 \tag{13}$$

orbital angular momentum

$$C = rv \cdot \sin\gamma \tag{14}$$

It was shown that the ΔV_c type of velocity correction would yield the optimal ΔV_{mc}. Figure 5 shows that angular deviation of nominal direction, $\delta\beta_2$, must be corrected. Therefore, partial derivatives with respect to β_2 are analyzed. It can be shown that $\partial e/\partial\beta_2 = \partial V_\infty/\partial\beta_2 = 0$ for $\beta_2 = 0$. Therefore,

$$\frac{\partial r_p}{\partial \beta_2} = \left[\frac{\dfrac{\partial e}{\partial \beta_2} v_\infty^2 - 2v_\infty \dfrac{\partial v_\infty}{\partial \beta_2}(e-1)}{v_\infty^4}\right]\mu = 0 \tag{15}$$

The r_p is a function of all six initial state variables. Its first partial with respect to any of them is zero for a direct hitting trajectory, as stated by Kizner and shown by Eq. (15). Therefore, the first linear term in Taylor expansion drops out and only the second quadratic term, with the second partial $\partial^2 r_p/\partial\beta_2^2$ and higher order terms remaining. This means that neglecting higher order terms δr_p must be represented as a quadratic function of all initial state vector errors. Therefore, Kizner justly considers that r_p is not a convenient measure of the target miss for a trajectory impacting a material point in the space.

The impact parameter B is determined by

$$B = C/v_\infty = (rv \sin\gamma)/h^{1/2} \tag{16}$$

For the nominal case

$$\partial B/\partial B_2 = [(\partial C/\partial\beta_2)\ v_\infty - (\partial v_\infty/\partial\beta_2)C]/v_\infty^2 \neq 0 \tag{17}$$

Therefore in the Taylor expansion for B about the nominal

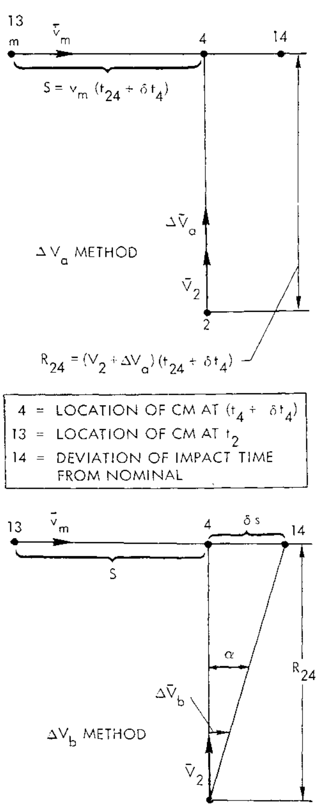


Fig. 4 Simplified rectilinear geometry of collision.

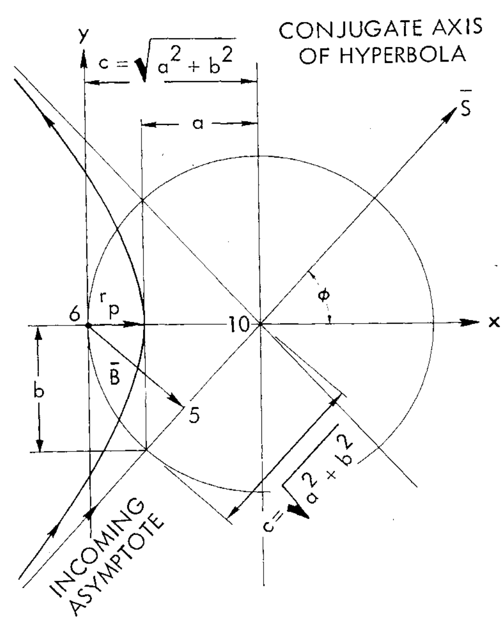


Fig. 6 Geometry of impact parameter.

Table 1 Impact data

No.	$TF = t_{14}$, min	Unbraked impact velocity V_4, m/sec	$-\gamma_4$ deg impact inertial	B, km	$-a$, km	$C = ae$, km	$\phi = \tan^{-1} (e^2 - 1)^{1/2}$, deg-min	DCA, km
1	3691.7	2676	74.46	1027.239	3227.982	3387.477	17–39	159.498
2	3676.6	2681	74.36	1021.406	3173.463	3333.786	17–50	160.314
3	3690.9	2681	74.14	1079.719	3171.478	3350.222	18–48	178.744
4	3699.7	2678	75.46	806.444	3199.990	3300.054	14–9	100.054
5	3696.9	2681	75.37	837.450	3172.543	3281.203	14–47	108.669
6	3721.5	2680	75.06	946.629	3178.334	3316.305	16–34	137.976
7	3727.6	2680	75.26	709.444	3176.617	3254.857	12–35	78.257
8	3740.5	2681	76.00	764.704	3175.929	3266.700	13–32	90.766
9	3753.2	2681	75.67	809.666	3173.641	3275.293	14–19	101.654
10	3769.4	2681	80.63	1326.686	3170.315	3436.717	22–43	266.398
11	3769.4	2681	80.51	1313.119	3168.213	3429.559	22–31	261.343
12	3772.4	2681	80.55	1342.644	3167.717	3440.521	22–59	272.795
13	3786.7	2681	80.19	1375.319	3168.118	3453.756	23–28	285.644
14	3810.5	2681	80.95	1368.520	3167.613	3450.607	23–22	282.984
15	3811.2	2681	80.98	1373.766	3168.973	3453.927	23–23	284.955
16	3811.4	2681	80.99	1367.448	3167.590	3450.139	23–21	282.561
17	3813.6	2681	80.83	1375.349	3166.593	3452.378	23–29	285.782
18	3821.8	2681	80.64	1394.558	3167.118	3460.551	23–46	293.435
19	3844.1	2681	80.47	1288.340	3167.961	3419.909	22–8	251.951
20	3845.4	2681	80.42	1290.973	3167.819	3420.769	22–9	252.954
21	3850.4	2681	80.31	1311.919	3166.956	3427.935	22–30	260.980
22	3768.0	2681	80.64	1244.200	3168.600	3435.810	22–45	267.147
23	3768.0	2681	80.53	1309.368	3165.257	3425.378	22–28	260.133
24	3768.0	2681	80.55	1312.449	3166.758	3427.952	22–31	261.197
25	3714.0	2688	69.38					
26	3882.0	2684	73.18					
27	3819.0	2651	51.56	1500.303	3536.640	3841.675		305.050
28	3746.0	2659	67.98				22–59	
29	3691.0	2659	66.92	17.789	3427.185	3427.219	0–18	151.134
30	3820.0	2681	80.79	1401.626	3168.359	3464.537	23–52	296.183
31		2681	80.63	1393.147	3166.646	3459.561	23–45	292.907

trajectory the second term can be neglected yielding the linearized version of

$$\delta B \cong (\partial B/\partial \beta_2)\, \delta\beta_2 \tag{18}$$

which has definite advantages over the δr_p.

The trajectory of interest in the neighborhood of the target body is a hyperbola. From the Fig. 6 we realize that geometrically the miss distance is

$$r_p = c - a \tag{19}$$

The equation of a hyperbola shown in Fig. 6 is

$$(x + c)^2/a^2 - y^2/b^2 = 1 \tag{20}$$

By definition

$$c = (a^2 + b^2)^{1/2} \tag{21}$$

Equation of the asymptote in normalized form is

$$\frac{b}{(a^2 + b^2)^{1/2}}\, x - \frac{a}{(a^2 + b^2)^{1/2}}\, y - \frac{bc}{(a^2 + b^2)^{1/2}} = 0 \tag{22}$$

The distance, B, from the center of the target is

$$|\bar{B}| = B = \frac{bc}{(a^2 + b^2)^{1/2}} = \frac{bc}{c} = b = (c^2 - a^2)^{1/2} = [(c - a)(c + a)]^{1/2} \tag{23}$$

At this place Kizner remarks that a can be neglected because

$$b \gg a \tag{24}$$

for interplanetary trajectories having large miss distance, such as 50 target diam (see Table 1 for comparison). In this case we can write that

$$|\bar{B}| = b \approx c \cong (c - a) = r_p = \text{target miss for a material point} \tag{25}$$

is approximately equal in magnitude to the real miss distance $r_p = DCA$.

4. Description of Standard Method

The injection state vector error, $\bar{\Delta}1$

$$\bar{\Delta}1 = [\delta x_1, \delta y_1, \delta z_1, \delta \dot{x}_1, \delta \dot{y}_1, \delta \dot{z}_1,]^T = [\delta \bar{p}_1,\ \delta \bar{v}_1]^T \tag{26}$$‡

causes a three-dimensional target miss vector, $\Delta\bar{M}$ expressed in both TRS and xyz coordinate systems (see Fig. 2). The errors in both systems are linearly related by matrix $[T]$

$$\delta\bar{M} = \begin{vmatrix} \delta\bar{B}\cdot\hat{T} \\ \delta\bar{B}\cdot\hat{R} \\ \delta\bar{T}F\cdot\hat{S} \end{vmatrix} = [\mathrm{T}]\cdot\ [\delta x_4, \delta y_4, \delta z_4, \delta \dot{x}_4, \delta \dot{y}_4, \delta \dot{z}_4]^T = [T]\ [T_{14}] \begin{vmatrix} \delta\bar{p}_1 \\ \delta\bar{v}_1 \end{vmatrix} \tag{27}$$

$$[T_{14}] = \begin{vmatrix} \dfrac{\partial x_4}{\partial x_1} & \dfrac{\partial x_4}{\partial y_1} & \cdots & \dfrac{\partial x_4}{\partial \dot{z}_1} \\ \dfrac{\partial y_4}{\partial x_1} & & \cdots & \dfrac{\partial y_4}{\partial \dot{z}_1} \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ \dfrac{\partial \dot{z}_4}{\partial x_1} & & \cdots & \dfrac{\partial \dot{z}_4}{\partial z_1} \end{vmatrix} \tag{28}$$

‡ Here T stands for transpose

Table 2 Injection and midcourse data[1,4]

No.	Launch Year	Launch Date	Time, sec	Az, deg	$\Delta \bar{V}_{mc} = FOM_m$,[a] m/sec	FOM_{m+T}, m/sec
1	1965	9–27	27,023	99.94	11.13	18.84
2	1965	9–27	28,500	106.09	11.72	19.41
3	1965	9–27	29,700	112.5	12.74	19.95
4	1965	9–28	30,300	93.49	9.02	19.56
5	1965	9–28	33,000	103.24	10.22	20.59
6	1965	9–28	34,523	114.0	11.65	20.88
7	1965	9–29	34,200	91.35	7.49	20.09
8	1965	9–29	37,800	106.72	9.23	21.10
9	1965	9–29	38,463	114.0	10.16	21.01
10	1965	9–30	37,800	92.71	7.71	20.00
11	1965	9–30	39,000	96.45	8.08	20.52
12	1965	9–30	40,200	101.87	8.59	20.77
13	1965	9–30	41,508	114.00	9.81	20.73
14	1965	10–1	39,296	92.40	7.14	19.39
15	1965	10–1	39,900	94.11	7.31	19.67
16	1965	10–1	41,400	99.13	7.81	20.29
17	1965	10–1	42,600	104.97	8.33	20.59
18	1965	10–1	43,735	114.0	8.64	19.95
19	1965	10–2	42,437	99.59	6.50	18.19
20	1965	10–2	43,800	105.16	6.96	18.71
21	1965	10–2	45,296	114.0	7.88	19.55
22	1965	9–30	31,776	90.00	7.6	20.02
23	1965	9–30	37,087	90.75	7.5	20.02
24	1965	9–30	37,500	91.86	7.7	20.24
25	1965	9–27	30,228	114.00	15.78	22.19
26	1965	10–3	48,162	114.00	9.42	21.57
27	1966	4–5	68,553	114.00	12.76	20.15
28	1966	4–7	77,605	114.00	11.08	20.60
29	1966	10–16	33,935	116.31	12.13	22.01
30	1965	10–1	43,500	111.72	8.2	19.63
31	1965	10–1	43,735		8.61	19.94

[a] Figure of merit (FOM) is the midcourse velocity necessary to correct the one-sigma injection error.

$$[T] = \begin{vmatrix} \frac{\partial \bar{B}\cdot\hat{T}}{\partial x_4} & \frac{\partial \bar{B}\cdot\hat{T}}{\partial y_4} & \frac{\partial \bar{B}\cdot\hat{T}}{\partial z_4} & \frac{\partial \bar{B}\cdot\hat{T}}{\partial \dot{x}_4} & \frac{\partial \bar{B}\cdot\hat{T}}{\partial \dot{y}_4} & \frac{\partial \bar{B}\cdot\hat{T}}{\partial \dot{z}_4} \\ \frac{\partial \bar{B}\cdot R}{\partial x_4} & & \cdots & & & \frac{\partial \bar{B}\cdot\hat{R}}{\partial \dot{z}_4} \\ \frac{\partial \bar{T}F\cdot\hat{S}}{\partial x_4} & & \cdots & & & \frac{\partial \bar{T}F\cdot\hat{S}}{\partial \dot{z}_4} \end{vmatrix} \tag{29}$$

To remove this target miss, a velocity impulse, $\Delta\bar{V}_{mc}$, is applied at point 2, yielding another target miss, $\delta\bar{M}_{mc}$, such that

$$\delta\bar{M} + \delta\bar{M}_{mc} = 0 \tag{30}$$

We can also write that

$$\delta\bar{M} = [T]\,[T_{24}] \begin{vmatrix} \delta\bar{p}^2 \\ \delta\bar{v}^2 \end{vmatrix} \tag{31}$$

$$[T_{24}] = \begin{vmatrix} \frac{\partial x_4}{\partial x_2} & \frac{\partial x_4}{\partial y_2} & \cdots & \frac{\partial x_4}{\partial \dot{z}_2} \\ \frac{\partial y_4}{\partial x_2} & & \cdots & \frac{\partial y_4}{\partial \dot{z}_2} \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ \frac{\partial \dot{z}_4}{\partial x_2} & & \cdots & \frac{\partial \dot{z}_4}{\partial \dot{z}_2} \end{vmatrix} \tag{32}$$

Since at point 2 only velocity but not position is corrected.

$$\delta\bar{M}_{mc} = [K_v]\cdot\Delta\bar{V}_{mc} \tag{33}$$

where $[K_v]$ is a matrix to be determined. We can also write that

$$\Delta\bar{V}_{mc} = \begin{vmatrix} \delta\dot{x}_{mc} \\ \delta\dot{y}_{mc} \\ \delta\dot{z}_{mc} \end{vmatrix} = [K_v]^{-1}\,\delta\bar{M}_{mc} = -[K_v]^{-1}\,\delta\bar{M} \tag{34}$$

From Eqs. (27, 31, and 33) it is evident that K_v is the velocity portion of the inverse of the matrix product $[T]\ [T_{24}]$ with the negative sign. Consequently

$$[K_v] = \begin{vmatrix} \frac{\partial \bar{B}\cdot\hat{T}}{\partial \dot{x}_{mc}} & \frac{\partial \bar{B}\cdot\hat{T}}{\partial \dot{y}_{mc}} & \frac{\partial \bar{B}\cdot\hat{T}}{\partial \dot{z}_{mc}} \\ \frac{\partial \bar{B}\cdot\hat{R}}{\partial \dot{x}_{mc}} & \frac{\partial \bar{B}\cdot\hat{R}}{\partial \dot{y}_{mc}} & \frac{\partial \bar{B}\cdot\hat{R}}{\partial \dot{z}_{mc}} \\ \frac{\partial \bar{T}F\cdot\hat{S}}{\partial \dot{x}_{mc}} & \frac{\partial \bar{T}F\cdot\hat{S}}{\partial \dot{y}_{mc}} & \frac{\partial \bar{T}F\cdot\hat{S}}{\partial \dot{z}_{mc}} \end{vmatrix} \tag{35}$$

Inserting (27) into (34) obtain

$$\Delta\bar{V}_{mc} = -[K_v]^{-1}\,[T][T_{14}] \begin{vmatrix} \delta\bar{p}_1 \\ \delta\bar{v}_1 \end{vmatrix} \tag{36}$$

This expression serves for computation of the midcourse correction.

5. ΔV_{mc} Variation

The standard method provides for the capability of correcting two-dimensional or three-dimensional miss. If only the impact parameter $\bar{B}$ is considered, the ΔV_{mc} is computed for "miss-only" correction. It is evident that the "miss-only" correction requires significantly smaller correction velocity (see Table 2). However, it was observed that the velocity corrections of the "miss-only" type show significant variation in the magnitude (in Table 2) (up to 100%) for the same space vehicle launched at different time. This variation required to correct nearly identical injection errors could not be explained solely by the variation of the inertial platform (IMU) orientation due to changes in the firing azimuth, or by variations in the distance to the moon. Reference 3 presents a theory of the variation of the differential correction and gives approximate expressions to estimate this variation. Reference 4 utilizes 25 simulated trajectories and error analyses to deduce another theory of the ΔV_{mc} variation. Both methods are of the descriptive nature. They observe and describe the behavior of the ΔV_{mc} and show how to estimate the necessary midcourse correction velocity in the future.

For impact trajectories, only the impact error, $\delta \bar{r}_4$, is of interest. However, in the standard method ΔV_{mc} is linearly related to $\delta \bar{B}$ and not to $\delta \bar{r}_4$. For a constant $\delta \bar{r}_4$, however, for the same type of trajectory, $\delta \bar{B}$ varies with time due to variation of the relative velocity at the moon. This also causes ΔV_{mc} variation, which presents a drawback of the method because it does not result from the real physical requirements of correction of $\delta \bar{r}_4$.

6. Nonuniqueness

Besides variability of ΔV_{mc} as the result of utilizing $\delta \bar{B}$ there is another drawback of the standard method. The impact error $\delta \bar{B}$ is presented as a two-dimensional vector in the plane perpendicular to the asymptote. This error, however, is not uniquely related to the to-be-corrected position error $\delta \bar{r}_4$. Its nonuniqueness has two different aspects.

1) The *TRS* coordinate system in which $\delta \bar{B}$ is expressed is not unique but time-varying with respect to another selenocentric rotating coordinate system in which $\delta \bar{r}_4$ is uniquely determined for a nonlibrating moon. Two axes of this other system lie in the lunar orbital plane, and one is directed toward the center of the Earth.

2) $\delta \bar{B}$ is not unique in the *TRS* coordinate system either, because it is a function of both $\delta \bar{r}_4$ and $\delta \bar{V}_4$. Therefore, different combinations of $\delta \bar{r}_4$ and $\delta \bar{V}_4$ may produce the same $\delta \bar{B}$. An explanatory numerical example is given in Sec. 8 of this paper. Correspondingly, sensitivities of $[K_v]$ computed for one specific trajectory during the preliminary studies cannot be used for midcourse correction during the actual flight.

7. Error Analysis

Certain assumptions made by W. Kizner at the introduction of the impact parameter are not always valid for a general type of impact trajectory. [See Eq. (24) and Table 1.] Therefore a question arises as to how much the method of the computation of the midcourse correction can be affected for a general case. To answer this question error analysis is presented in this portion of the paper. According to the original idea, the impact parameter $\bar{B}$ presents the target miss. The standard (nominal) trajectory is postulated as that going through the c.m. (Refs. 2 and 3). This implies that for a nominal standard trajectory $\bar{B} = 0$. However, there are many trajectories which must be called nominal, since they satisfy all constraints, yet which yield certain impact parameter of nonzero magnitude for $\Delta \bar{4} = 0$ (Tables 1 and 2). In order to preserve the method utilizing $\bar{B}$ for computation of $\Delta \bar{V}_{mc}$ such an impact parameter is interpreted as the "nominal" impact parameter and is designated by $\bar{B}_n$, or, simply $\bar{B}$. Another "perturbed" impact parameter, $\bar{B}_p$, would correspond to $\Delta \bar{4} \neq 0$. The standard method of $\Delta \bar{V}_{mc}$ computation, utilizes $\delta \bar{B}$ which is interpreted as the target miss, $\delta \bar{M}$

$$\delta \bar{M} = \delta \bar{B} = \bar{B}_p - \bar{B}_n \tag{37}$$

We designate by $\hat{B}$ the unit vector along $\bar{B}_n$ such that

$$\bar{B}_n = B_n \hat{B} \tag{38}$$

The assumption made by the standard method is that the direction of $\bar{S}_p$ is the same as that of $\bar{S}_n$ in all practical cases. Accordingly, both $\bar{B}_n$ and $\bar{B}_p$ lie in the same plane. Therefore the selenocentric coordinate system is the same;

$$TRS = T_n R_n S_n = T_p R_p S_p \tag{39}$$

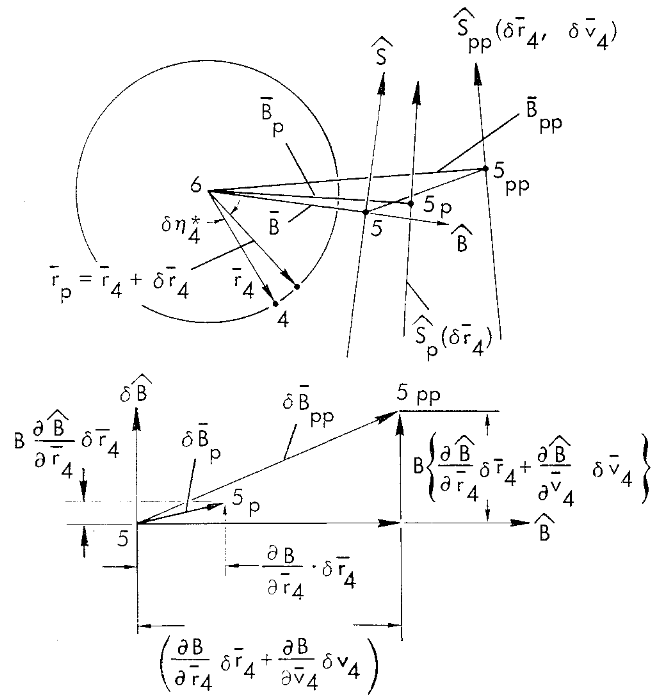


Fig. 7 $\delta \bar{B}$ as function of different perturbations.

(If perturbation is limited to the nominal trajectory plane, then $\hat{B}_n = \hat{B}_p$.)

Therefore,

$$\delta \bar{M} = \delta \bar{B} = \begin{Bmatrix} \bar{B}_{pT} \cdot \hat{T} \\ \bar{B}_{pR} \cdot \hat{R} \\ 0 \end{Bmatrix} - \begin{Bmatrix} \bar{B}_{nT} \cdot \hat{T} \\ \bar{B}_{nR} \cdot \hat{R} \\ 0 \end{Bmatrix} \tag{40}$$

In this way, the two-dimensional miss is measured by

$$\delta \bar{M} = \delta B_T \hat{T} + \delta B_R \hat{R} \tag{41}$$

We see that

$$\bar{B} = \begin{vmatrix} \bar{B}_T \\ \bar{B}_R \\ \bar{B}_S \end{vmatrix} = \begin{vmatrix} B_T \hat{T} \\ B_R \hat{R} \\ B_S \hat{S} \end{vmatrix} = [Q] \begin{vmatrix} x_4 \hat{\imath} \\ y_4 \hat{\jmath} \\ z_4 \hat{k} \end{vmatrix} = [Q] \bar{r}_4 \tag{42}$$

where Q is transformation mapping impact point 4 from SCI into the nominal *TRS* system; the dependence of δQ and δB on the $\Delta \bar{4}$ is shown on Fig. 7. (The nonlinear transformation $[Q(\bar{r}_4, \bar{V}_4)]$ involves certain operations on $\bar{r}_4$. See Ref. 1, Appendix J. The $[\delta Q] = [\delta Q(\delta \bar{r}_4, \delta \bar{V}_4)]$.) We can now perturb both sides of Eq. (42) and analyze the problem in its most general form, avoiding any simplifying assumptions;

$$\begin{vmatrix} \delta B_T \hat{T} + B_T \delta \hat{T} \\ \delta B_R \hat{R} + B_R \delta \hat{R} \\ \delta B_S \hat{S} + B_S \delta \hat{S} \end{vmatrix} = [\delta Q] \begin{vmatrix} x_4 \hat{\imath} \\ y_4 \hat{\jmath} \\ z_4 \hat{k} \end{vmatrix} + [Q] \begin{vmatrix} \delta x_4 \hat{\imath} & x_4 \delta \hat{\imath}^0 \\ \delta y_4 \hat{\jmath} + & y_4 \delta \hat{\jmath}^0 \\ \delta z_4 \hat{k} & z_4 \delta \hat{k}^0 \end{vmatrix} \tag{43}$$

We can rearrange Eq. (43) as

$$\delta \bar{r}_4 = \begin{vmatrix} \delta x_4 \hat{\imath} \\ \delta y_4 \hat{\jmath} \\ \delta z_4 \hat{k} \end{vmatrix} = [Q]^{-1} \begin{vmatrix} \delta B_T \hat{T} \\ \delta B_R \hat{R} \\ \delta R_S \hat{S} \end{vmatrix} + [Q]^{-1} \begin{vmatrix} B_T \delta \hat{T} \\ B_R \delta \hat{R} \\ B_S \delta \hat{S} \end{vmatrix} - [Q]^{-1}[\delta Q] \begin{vmatrix} x_4 \hat{\imath} \\ y_4 \hat{\jmath} \\ z_4 \hat{k} \end{vmatrix} \tag{44}$$

Equation (44) can be rewritten in the form of

$$\delta \bar{r}_4 = [Q]^{-1} \delta B \hat{B} + [Q]^{-1} B \delta \hat{B} - [Q]^{-1} [\delta Q] \bar{r}_4 \tag{45}$$

It is evident that we desire to correct only the impact position error $\delta \bar{r}_4$, as given by Eq. (45). There is no intention to correct $\delta \bar{v}_4$. Beside that, this would be impossible with one single impulse of $\Delta \bar{V}_{mc}$. After some manipulation with Eq.

(45) we obtain

$$\delta \bar{r}_4 = \left[\frac{\partial B\hat{B}}{\partial \bar{r}_4}\right]^{-1}\{[Q]\delta\bar{r}_4 + [\delta Q]\bar{r}_4 - B\delta\hat{B}\} - \left[\frac{\partial B\hat{B}}{\partial \bar{r}_4}\right]^{-1}\left[\frac{\partial B\hat{B}}{\partial \bar{v}_4}\right]\delta\bar{v}_4 \quad (46)$$

The $\delta\bar{r}_4$ given by (46) is the position impact error, which we desire to correct only. We can say that the total $\delta\bar{B}$ which should be considered for $\Delta\bar{V}_{mc}$ computation is

$$\left[\frac{\partial B\hat{B}}{\partial r_4}\right]\delta\bar{r}_4 + B\left[\frac{\partial\hat{B}}{\partial r_4}\right]\delta\bar{r}_4 = \{[Q]\,\delta\bar{r}_4 + [\delta Q]\,\bar{r}_4\} - B\left[\frac{\partial\hat{B}}{\partial v_4}\right]\delta\bar{v}_4 - \left[\frac{\partial B\hat{B}}{\partial v_4}\right]\delta\bar{v}_4 \quad (47)$$

From Eq. (47) we can see that usually two errors are committed during $\Delta\bar{V}_{mc}$ computation: a) The error $\delta\bar{B}_1$ due to inclusion of $\delta\bar{v}_4$ into computation of $[T]$ as per (29) is

$$\delta\bar{B}_1 = -\left\{\left[\frac{\partial B\hat{B}}{\partial\bar{v}_4}\right]\delta\bar{v}_4 + B\left[\frac{\partial\hat{B}}{\partial\bar{v}_4}\right]\delta\bar{v}_4\right\} \quad (48)$$

b) The error $\delta\bar{B}_2 = B\delta\hat{B}$ along the $\bar{S}$, due to neglection of the change in the orientation of $\bar{S}$ due to total $\Delta\bar{4}$ is

$$\delta\bar{B}_2 = B\left[\frac{\partial\hat{B}}{\partial\bar{r}_4}\right]\delta\bar{r}_4 \quad (49)$$

The impact position error to be corrected, $\delta\bar{r}_4 = [\delta x_y, \delta y_y, -\delta z_y]^T$ is expressed in the inertial SCI, coordinate system. Therefore, rewrite Eq. (47) by expressing $\delta\bar{B}_1$ and $\delta\bar{B}_2$ in inertial circular coordinate system (see Appendix A), remembering that for a spherical moon which we consider here, $\delta r_4 = 0$. We assume here for simplicity, that the state vector error, $\Delta\bar{4}$, is limited to the nominal trajectory plane;

$$\frac{\partial B\hat{B}}{\partial\eta_4{}^*}\,\delta\eta_4{}^* = \{[Q]\delta\bar{r}_4 + [\delta Q]\bar{r}_4\} - B\left\{\frac{\partial\hat{B}}{\partial\eta_4{}^*}\,\delta\eta_4{}^*\right\} - B\left\{\frac{\partial\hat{B}}{\partial v_4}\,\delta v_4 + \frac{\partial\hat{B}}{\partial\gamma_4{}^*}\,\delta\gamma_4{}^*\right\} - \left\{\frac{\partial B\hat{B}}{\partial v_4}\,\delta v_4 + \frac{\partial B\hat{B}}{\partial\gamma_4{}^*}\,\delta\gamma_4{}^*\right\} \quad (50)$$

or using (48) and (49) we can write

$$\frac{\partial B\hat{B}}{\partial\eta_4{}^*}\,\delta\eta_4{}^* = \{[Q]\delta\bar{r}_4 + [\delta Q]\bar{r}_4\} - B\delta\hat{B} - \delta B\hat{B}_0 = \{[Q]\delta\bar{r}_4 + [\delta Q]\bar{r}_4\} - \bar{C} \quad (51)$$

where

$$\bar{C} = \delta B'\hat{B} + B\delta\hat{B} = \delta\bar{B}_1 + \delta\bar{B}_2 \quad (52)$$

It is clear from (51) that the present method would work ideally for those cases in which $\bar{C} = 0$. We are going to see when such cases would occur.

Impact at the Material Point

In this case it can be shown[1] that $\delta\bar{B}_1$ reduces to $(\partial\bar{B}/\partial\gamma_4{}^*)\text{-}\delta\gamma_4{}^*$. It is also clear that $\delta\bar{B}_2 = 0$. Therefore, Eq. (51) reduces to

$$\frac{\partial\bar{B}}{\partial\eta_4{}^*}\,\delta\eta_4{}^* = \{[Q]\delta\bar{r}_4 + [\delta Q]\bar{r}_4\} - \frac{\partial\bar{B}}{\partial\gamma_4{}^*}\,\delta\gamma_4{}^* \quad (53)$$

or

$$\delta\bar{B} = \left\{\frac{\partial\bar{B}}{\partial\eta_4{}^*}\,\delta\eta_4{}^* + \frac{\partial\bar{B}}{\partial\gamma_4{}^*}\,\delta\gamma_4{}^*\right\} = \{[Q]\delta\bar{r}_4 + [\delta Q]\bar{r}_4\} \quad (54)$$

However, the total δB will be corrected ideally, as shown by (54) since $\delta\bar{B}$ is equal to $\delta\{[Q]\bar{r}_4\}$.

We see that in the impact case of the material point trajectory we must correct both position error $\delta\eta_4{}^*$ and velocity direction error $\delta\gamma_4{}^*$, in order to pass through the c.m. In this respect the material point impact case differs from all other impact trajectories, including the standard trajectory. It becomes evident that B-method of $\Delta\bar{V}_{mc}$ computation is completely adequate for this case.

Standard Impact at the Planetary Surface

Reference 1 shows that sensitivities in the first approximation are the same for a standard trajectory as for the material-point impact trajectory. Therefore, general conclusions for this case are similar with one single exception, namely: this time we do not have, neither do we like to correct the portion of δB corresponding to $(\partial B/\partial\gamma_4{}^*)\delta\gamma_4{}^*$ as it was the case with the material-point impact trajectory. Therefore, the complete correction of $\delta\bar{B}$ in this case would not be ideal.

$C = 0$ in a General Case

Equation (51) for $\bar{C} = \bar{0}$, reduces to

$$\delta\bar{B} = \delta\bar{B}(\delta\bar{r}_4) = \delta\{[Q]\bar{r}_4\} \quad (55)$$

To make $\bar{C} = \bar{0}$ we must satisfy

$$\bar{C} = \delta B'\hat{B} + B\delta\hat{B} = \bar{0} \quad (56)$$

Since, however, $\delta B'\hat{B}$ and $B\delta\hat{B}$ are mutually perpendicular, the only possible way that (56) can be satisfied is if

$$\delta B'\hat{B} = B\delta\hat{B} = \bar{0} \quad (57)$$

The $B\delta\hat{B} = 0$ is satisfied if $B = \bar{0}$, which occurs for a material-point impact or a standard trajectory. To satisfy $\delta B'\bar{B} = 0$ we must satisfy $\delta v_4 = \delta\gamma_4{}^* = 0$ for a general case, since both sensitivities $(\partial B\hat{B}/\partial v_4,\ \partial B\hat{B}/\partial\gamma_4{}^*)$ are not zero at once, even for the material point case, as was shown in Ref. 1. Therefore, $\bar{C} = \bar{0}$ only if both following conditions are satisfied: 1) trajectory is standard in which case $B = \partial B/\partial v_4 = 0$ (Ref. 1), and 2) $\delta_{\gamma_4{}^*} = 0$. This means that generally, even for a standard trajectory $\bar{C} \neq \bar{0}$.

Fly-by Trajectory

If no surface impact is required but only a magnitude of the perifocal distance and velocity, combined with controlled orbital inclination, the impact parameter is OK. In this case the problem is the same as that of the material-point trajectory. With the exception that only partial correction is required.

7. Impact Time Dependent Error Source

For the nominal case $[Q]$ is unique, therefore $\bar{B} = B\hat{B}$ is a vector fixed in the inertial space *SCI*. Therefore, the B vector which we will designate by $B_0(t_0)$ presents another inertial reference. The t_0 in the brackets indicates the impact time of the nominal trajectory. Therefore, we can rewrite expression (52) as $\bar{C}_0 = \delta B'\hat{B}_0(t_0) + B\delta\hat{B}_0(t_0)$. For a trajectory presenting only a slightly perturbed version of the nominal characterized by $B_0(t_0)$, the second term of the preceding expression, $B\delta B_0(t_0)$ will be small. Consider now several other nominal trajectories impacting the same point on the moon but spread within a lunar month. (We assume zero libration. See Fig. 8.) Their nominal impact parameters are oriented along the corresponding different unit vectors, $B_1(t_1)$, $B_2(t_2)$ ... $B_n(t_n)$, etc.;

$$\bar{B}_0(t_0) = [I]\bar{B}_0(t_0)$$
$$\cdots\cdots\cdots\cdots$$
$$\bar{B}_n(t_n) = [U_{0n}]\bar{B}_0(t_0)$$

In these expressions transformation $[U_{0n}]$ presents the orthogonal transformation between $(TRS)_0$ and $(TRS)_n$ coordinate systems, and besides that the appropriate change in the magnitude of $|\bar{B}_n|$ as compared to $|\bar{B}_0|$. Utilizing expression (42) we can generally write

$$\bar{B}_n(t_n) = [U_{0n}]\,[Q]\,\bar{r}_4(t_0)$$

In this expression on the right side only the transformation $[U_{0n}]$ changes within the lunar month because of the changing relative geometry of the earth and the moon. Therefore, the nominal $\bar{B}$ will vary during the lunar month. (For the same type of trajectories, e.g., constant-impact velocity trajectories.) Because of that it is to be expected that the corresponding ΔV_{mc} will also change within a lunar month. For the nominal cases

$$B_0(t_0) = B_1(t_1) = B_2(t_2) = \ldots B_n(t_n) = 0$$

However, for similar injection perturbations different errors would result as follows: (I is the identity matrix.)

$$\bar{C}_0 = [I]\{\delta B'_0\hat{B}_0(t_0) + B_0\delta\hat{B}_0(t_0)\} + [\delta I]B_0(t_0) = [I]\;\bar{C}_0$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$\bar{C}_n = [U_{0n}]\bar{C}_0 + [\delta U_{0n}]\bar{B}_0(t_0)$$

This means that the time dependent portion of the error, $[\delta U_{0n}]\bar{B}_0(t_0)$ will appear each time when the impact time differs from the original impact time, t_0 within a lunar month.

If the inertially fixed $\hat{B}_0$ direction is used as the reference, then the error $\bar{C}_n$ can be presented as

$$\bar{C}_n = \{\delta B'_0 + [\delta U_{0n}]B_0(t_0)\cdot\hat{B}_0(t_0)\}\hat{B}_0 + \{B_0 + [\delta U_{0n}]B_0(t_0)\cdot\delta\hat{B}_0(t_0)\}\,\delta\hat{B}_0(t_0)$$

It can be seen from this expression that the second term on the right side of this expression, which is the error along the direction normal to B_0, can become large depending on a particular case. This second term is strongly contributing to ΔV_{mc} variation discussed in Refs. 1 and 3. The time variation for standard trajectory was especially discussed in Ref. 3. Since the aforementioned variation of B depends on the lunar motion and on the nominal trajectory, it is predictable. As such it can and should be compensated.

8. Numerical Example

Assume that the impact error is $\Delta\bar{4} = \delta\gamma_4{}^* = 1°$ and using appropriate numerical values compute (See Ref. 1, Appendix, Table 3c)

$$C_p = r_p v_p \sin\gamma_4{}^* = 1738 \times 2800 \times 0.01745 = 84.9187\ \text{km}^2/\text{sec}$$

$$h_p = v_{4_p}{}^2 - 2\mu/r_{4_p} = 2.8^2 - 2(4890)/1738 = 7.84 - 5.627 = 2.213\ \text{km/sec}$$

$$v_{\infty p} = (h_p)^{1/2} = 1.4876\ \text{km/sec}$$

$$\delta B = 84.9187/1.4876 = 57.08\ \text{km}$$

For $\Delta\bar{4} = \delta\eta_4{}^* = 1°$ obtain

$$r_4\delta\eta_4{}^* = 30.328\ \text{km}$$

$$C'_p = 30.328 \times 2.8 = 84.9187\ \text{km}$$

$$\delta B'_p = B'_p = C'_p/v_\infty = 84.9187/1.4876 = 57.08\ \text{km}$$

Because both angular errors, $\delta\gamma_4{}^*$ and $\delta\eta_4{}^*$ are in the plane of nominal trajectory, both $\bar{S}$'s and $\bar{B}$'s also lie in this plane. Both $\bar{S}$'s are parallel, according to the basic assumption of the standard method.

In both cases δB has the same magnitude and orientation, being perpendicular to the same direction of asymptote; accordingly, $\bar{S}_n = \bar{S}_p$. Therefore, the method will compute the same ΔV_{mc} in both cases although it is clear, that the first case needs no correction.

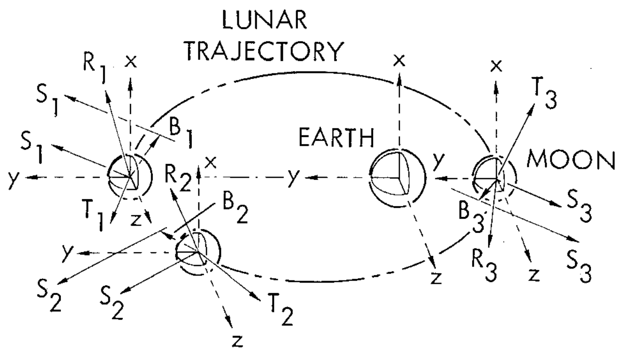


Fig. 8 Variation of *TRS* system with time.

9. Possible Improvements and Normalization

A partial improvement of the standard method can be achieved if impact velocity error, $\delta\bar{v}_4$, is excluded from the computation of $\bar{B}$. This can be done by replacing expression (29) by a new one;

$$[T_n] = \begin{bmatrix} \dfrac{\partial\bar{B}\cdot\hat{T}}{\partial x_4} & \dfrac{\partial\bar{B}\cdot\hat{T}}{\partial y_4} & \dfrac{\partial\bar{B}\cdot\hat{T}}{\partial z_4} \\ \dfrac{\partial\bar{B}\cdot\hat{R}}{\partial x_4} & \dfrac{\partial\bar{B}\cdot\hat{R}}{\partial y_4} & \dfrac{\partial\bar{B}\cdot\hat{R}}{\partial z_4} \end{bmatrix} \tag{58}$$

The general effect of this modification will be that the usual probable over-correction of $\delta\bar{r}_4$ will be avoided. See Ref. 1 for detailed discussion, also numerical examples. Instead, the undercorrection of $\delta\bar{r}_4$ may take the place.

It can be shown that if the impact location is specified by two angles, (right ascension, α_4 and declination, δ_4 or, longitude, λ_4 and latitude, ϕ_4) then for small injection errors expected for impact trajectories

$$\delta\bar{M} = \begin{vmatrix}\delta\alpha_4 \\ \delta\delta_4\end{vmatrix} = \begin{vmatrix} \dfrac{\partial\alpha_4}{\partial\dot{x}_{mc}} & \dfrac{\partial\alpha_4}{\partial\dot{y}_{mc}} & \dfrac{\partial\alpha_4}{\partial\dot{z}_{mc}} \\ \dfrac{\partial\delta_4}{\partial\dot{x}_{mc}} & \dfrac{\partial\delta_4}{\partial\dot{y}_{mc}} & \dfrac{\delta\alpha_4}{\partial\dot{z}_{mc}} \end{vmatrix} \begin{vmatrix}\Delta\dot{x}_{mc} \\ \Delta\dot{y}_{mc} \\ \Delta\dot{z}_{mc}\end{vmatrix} \tag{59}$$

$$\delta\bar{M} = \begin{vmatrix}\delta\alpha_4 \\ \delta\delta_4\end{vmatrix} = \begin{vmatrix} \dfrac{\partial\alpha_4}{\partial x_1} & \dfrac{\partial\alpha_4}{\partial y_1} & \dfrac{\partial\alpha_4}{\partial z_1} & \dfrac{\partial\alpha_4}{\partial\dot{x}_1} & \dfrac{\partial\alpha_4}{\partial\dot{y}_1} & \dfrac{\partial\alpha_4}{\partial\dot{z}_1} & \dfrac{\partial\alpha_4}{\partial t_1} \\ \dfrac{\partial\delta_4}{\partial x_1} & \dfrac{\partial\delta_4}{\partial y_1} & \dfrac{\partial\delta_4}{\partial z_1} & \dfrac{\partial\delta_4}{\partial\dot{x}_1} & \dfrac{\partial\delta_4}{\partial\dot{y}_1} & \dfrac{\partial\delta_4}{\partial\dot{z}_1} & \dfrac{\partial\delta_4}{\partial t_1} \end{vmatrix} \begin{vmatrix}\delta x_1 \\ \delta y_1 \\ \delta z_1 \\ \delta\dot{x}_1 \\ \delta\dot{y}_1 \\ \delta\dot{z}_1 \\ \delta t_1\end{vmatrix} \tag{60}$$

This indicates that, generally, the preceding angular presentation of target miss is more adequate than the δB presentation of impact trajectories. In order to compare $\Delta\bar{V}_{mc}$ for different trajectories they should be first normalized. Let's call the $\Delta\bar{V}^*$ the normalized $\Delta\bar{V}_{mc}$

$$\Delta\bar{V}^* = F(\Delta\bar{V}_{mc}) \tag{61}$$

i.e., $\Delta\bar{V}^*{}_{mc}$ is computed for some "normal" R_0, Az_0, TF_0, and Q_0. We can write a Taylor expansion for $\Delta\bar{V}_{mc}$ for a real case as,

$$\Delta\bar{V}_{mc}|R_0 + \delta R,\, Az_0 + \delta Az,\, TF + \delta TF,\, [Q_0 + \delta Q_0]| = \Delta\bar{V}^*{}_{mc}|R_0, Az_0, TF_0, Q_0| + \left\{\frac{\partial(\Delta V_{mc})}{\partial R}\,\delta R + \frac{\partial(\Delta V_{mc})}{\partial Az}\,\delta Az + \frac{\partial(\Delta V_{mc})}{\partial TF}\,\delta TF + \frac{\partial(\Delta V_{mc})}{\partial Q_0}\,\delta Q_0\right\} + \frac{\partial^2(\Delta V_{mc})}{\partial Q_0{}^2}\,\delta Q_0{}^2 + \text{other higher order terms} \tag{62}$$

Rearranging, write

$$q_i = \frac{\Delta V^*_{mc}}{\Delta V_{mc}} \cong 1 - \frac{1}{\Delta V_{mc}} \left\{ \frac{\partial(\Delta V_{mc})}{\partial R}\, \delta R + \frac{\partial(\Delta V_{mc})}{\partial Az}\, \delta Az + \frac{\partial(\Delta V_{mc})}{\partial TF}\, \delta TF + \frac{\partial(\Delta V_{mc})}{\partial Q_0}\, \delta Q_0 \right\} + \frac{\partial^2(\Delta V_{mc})}{\partial Q_0{}^2}\, \delta Q_0{}^2 + \ldots \tag{63}$$

where R = earth distance, A_2 = launch azimuth, TF = time of flight.

Since the Q_0 function is strongly nonlinear, it is presented by the first two terms of the Taylor's expansion. In Eq. (63), q is the normalization factor, such that to compare ΔV_{mc} for two real trajectories, we must first compute $\Delta V_{mc}{}^*$'s and only then compare them. In other words we can compare only $\Delta V_{mc}{}^*{}_1 = \Delta V_{mc1} \cdot q_1$, $\Delta V_{mc}{}^*{}_2 = \Delta V_{mc2} \cdot q_2$.

10. Conclusions

The following was shown:

1) The standard B method was introduced as a method applying one single correction for lunar trajectories to preserve the desired linearity between the target errors.

2) The material point target is the only one which presents an ideal case for $\delta \bar{B}$ approximation. All higher order terms will be zero. (See Ref. 1.)

3) Standard impact trajectory with $\delta\gamma_4{}^* = 0$ will be nearly as good, except that the higher order terms will be present. For larger perturbations the contribution may become significant. It is presumed that the impact time dependent error is compensated.

4) For all other impact trajectories, there may be first-order errors in ΔV_{mc} computed by the B-method. The error magnitude depends on each particular case.

5) The error due to change in B orientation may become significant if different trajectories are compared. Therefore it has to be compensated.

6) One general conclusion developed in Chap. 3 of Ref. 1 may be stated as follows. The midcourse correction velocity, ΔV_{mc}, a function of the target miss, is computed by the standard method as the first term of the Taylor expansion which presents the target miss. However, instead of using the true target miss as measured from the nominal impact point on the spherical lunar surface, the standard method measures the miss from the c.m. as the impact parameter B. This corresponds to the replacement of the nominal point of the expansion by another one. If the shifting of the point of expansion (were) small as compared to considered impact error, this replacement would be harmless. However, the lunar radius for which the expansion point is shifted is much larger than considered impact errors; therefore, depending on geometry of impact the standard method may significantly distort the computation of the midcourse correction velocity.

7) The quality of the B-method can be somewhat improved if only position error at impact, $\delta \bar{r}_4$, would be used for $[K_v]$ matrix computation.

8) "Right ascension," α_4, and "declination," δ_4, were chosen as the measure of the target miss, as measured from the nominal impact point, since they are more adequate for that purpose (see item 11).

9) There is a one-to-one relation between the $\Delta \bar{V}_{mc}$ as computed by means of sensitivities of Eq. (59) and the target miss.

10) Uniqueness of sensitivities mentioned in 9 makes it possible to utilize them both during preliminary studies and during the actual flight. This is a very desirable feature.

11) This paper is concerned only with impact trajectories which, generally, require good precision. Therefore, terminal errors for impact trajectories are expected to be much smaller than for fly-by trajectories for which the B-method, as previously shown, is adequate.

12) More detailed discussion and numerical data can be found in Ref. 1.

Appendix: Coordinate Systems

ECI, Geocentric, x,y,z

Inertial, right-handed orthogonal system with the origin at the center of the earth and inertially fixed orientations.

SCI, Selenocentric, x,y,z

Inertial, right-handed, orthogonal system with the origin at the center of the moon and axes parallel to ECI.

Target Centered, *TRS*

Right-handed, orthogonal system with the origin at the center of the target body and oriented such that the positive S axis points in the direction of the velocity vector from infinity, V_∞; the T axis is the intersection of the moon's orbit plane and the plane which contains the center of the target body and which is normal to the S axis, the positive sense of which is defined by $\bar{T} \equiv \bar{S} \times \bar{W}$ where $\bar{W}$ points in the direction normal to the moon's orbit plane; and the positive R axis completes the right-handed system through the relationship, $\bar{R} \equiv \bar{S} \times \bar{T}$ as defined in Fig. 2.

Cylindrical Coordinate System

Cylindrical coordinate system is an orbital coordinate system with the origin at the center of the moon. For geocentric zone: φ(true anomaly), R, N, V, β (angle between $\bar{R}$ and $\bar{V}$), $\dot{N}$. For selenocentric zone: η(true anomaly), r, n, v, γ (angle between $\bar{r}$ and $\bar{v}$), $\dot{n}$. Here N, $\dot{N}$, n, and $\dot{n}$ stand for position and velocity normal to the nominal motion plane.

Circular Coordinate System

This is a planar selenocentric inertial coordinate system, located in the nominal trajectory plane.

The coordinates are

$$r = |\bar{r}|$$

$$\eta^* = \cos^{-1}(\bar{r}_p \cdot \bar{r}_n)$$

$$v = |\bar{v}|$$

$$\gamma_p{}^* = \cos^{-1}(\bar{r}_n \cdot \bar{v}_p)$$

where subscripts n and p stand for "nominal" and "perturbed."

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